SHARING A PATH WITH TREBILCOCK: PREFERENCE, PLURALISM, AND PROPORTIONALITY

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I Pain and Preference

I first met Michael Trebilcock in the fall of 1978 when he invited me to come to the Faculty of Law at the University of Toronto as a Visiting Fellow in Law and Economics. It was my first academic position after finishing my doctoral research at Cambridge University and the beginning of a long and very happy association with Michael and the Faculty. Both have been incredibly good to me over the last thirty years or so, tolerating my flights of theoretical fancy in a way that they have never really deserved. In this short paper I ask both, the man and the institution, for a further such indulgence.

I came from Cambridge to Toronto armed with the weapons of logical impossibility. I had spent the last several years thinking about Kenneth Arrow's famous impossibility theorem where he shows that there exists no way of aggregating individual rankings of social states into a "social ranking" that has certain apparently desirable properties.[1] I will say a little more about the methods of social choice theory before I'm done, but for the moment I just want to emphasize again how tolerant Michael was of this peculiar interest of mine. While the theory of social choice had successfully come to dominate the pages of some economics journals over the 1970s,[2] it is fair to say, I think, that by 1978 the theory had made almost no inroads into law and economics.

This is probably easy enough to explain. The largely set theoretic methods of social choice theory bore almost no resemblance to methods otherwise being trotted out in economics at the time. It would be expecting a lot of anyone, already burdened with explaining the novel techniques of an unfamiliar discipline (economics) to new converts (legal academics), also to demonstrate the use of another set of tools that had yet to make any broad impact within the discipline. Further, there was the overtly normative nature of social choice theory, something that has never endeared it to those economists who so often claim to be interested only in empirically testable hypotheses. Finally, even for those prepared to engage explicitly in the analysis of values, the very nature of *logical* impossibility between values gives the whole field an "all-or-nothing" flavor, something that legal philosophers are more open to in their analyses of (categorical) rules and (uncompromised) principles, but something that is quite foreign to the lawyer-economist more used to the idea of price-sensitive "tradeoffs".[3]

But Michael tried very hard to look past all these obstacles and to see something in what I was doing. I'm not sure whether he has ever really found anything useful there over the entire three decades

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[1] Kenneth Arrow, *Social Choice and Individual Values* Second Edition (New Haven: Yale University Press, 1963)

[2] The *Review of Economic Studies* was receiving so many submissions of new impossibility results at one point in the 1970s that the editors had to call for a brief stay on submissions from social choice theorists.

[3] For good discussion, and critique, of this "all or nothing" character of social choice theory, see Michael Baurmann and Geoffrey Brennan, "Majoritarian Inconsistency, Arrow Impossibility, and the Comparative Interpretation: A Context-based View" in Christoph Engel and Lorraine Dalston, eds. *Is There Value in Inconsistency?* (Baden-Baden: Nomos, 2006) 93-117.

that I have known him, but his effort has been constant and valiant. And it was strikingly so in 1978 if one considers the two arguments that particularly engaged me then, and which I pressed upon him, somewhat relentlessly.

One of my arguments was that it was simply wrong to think that we should always require transitivity in social choice, that is, that if some alternative social choice A was preferred to another one B, and B was preferred to yet a third C, then (as transitivity, and some would say 'sanity', would demand) A should be preferred to C. Of course, I had my reasons for championing *intransitivity*, and still do, and I shall come back to them towards the end of this paper. But you can probably anticipate the effect of denying transitivity on any good rational *maximizer*. For if alternatives for choice cannot be ordered transitively, there simply is no best alternative to be found; for every alternative chosen, no matter which one, there will *always* be another which is preferred.[4] This is the famous cycling problem, and one can only imagine what Michael must have thought when I tried to suggest to him that this was really no problem at all. Yet he kept me around the Faculty for a full year.

My second argument was inspired by an article that I had read in *Philosophy and Public Affairs* over the previous year called "Should the Numbers Count?" by someone called John Taurek. (Taurek seems largely to have disappeared from the academic scene after publishing this article; not everyone is as puzzled by this as I am.) Taurek had argued that in a choice between saving one person and saving five persons from some calamity, say, some amount of pain or a certain death, the greater number of lives saved from this calamity should not be a factor.[5] All other things equal, one should flip a coin between saving the one and saving the five; that would give each and every person a fair and equal chance of being saved. But I was intrigued by Taurek's argument, and the examples he used, less because of the fair and equal chance argument and more because they gave a special life to the idea that there was something odd about the aggregation of a value (e.g., the avoidance of pain or death) which had no significance, *as an aggregation*, for any one individual in particular. There simply is no more pain, for example, than the pain that is suffered by some individual. So, quantity of pain being equal, the pain suffered by the one is a perfect match for the pain suffered by any one of the five individuals; the numbers of individuals suffering that pain simply do not count.[6] Of course, Michael would always advance the sensible question at this point: "What if it was a matter of either saving one or saving a thousand (or even a million)? Surely those numbers make a difference!" "The numbers don't count," I would chant in reply, with all the zeal of a recent convert. Still, Michael kept me around.

As it happens, the intransitivity argument and the number insensitivity argument are logically connected. It turns out that if one adds a couple of other reasonable choice conditions, namely, that the

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[4] Strictly, the requirement is one of acyclicity, not full transitivity; for discussion, see Amartya K. Sen, *Collective Choice and Social Welfare* (Edinburgh: Oliver and Boyd, 1970), at 16. For an indication of my disagreements with the idea that transitivity, or even acyclicity and maximization, were necessary to an understanding of how collective rationality needs to operate in social choice, see Bruce Chapman, 'Rights As Constraints: Nozick versus Sen' (1983) 15 Theory and Decision 1-10

[5] John Taurek, 'Should the Numbers Count?' (1977) 6 Philosophy and Public Affairs 293-316. While Taurek seems never to have revisited this argument, it has received a good deal of attention. For recent discussion, see Iwao Hirose, 'Aggregation and Numbers' (2004) Utilitas 62; and Nien-He Hsieh, Alan Strudler, and David Wasserman, 'Pairwise Comparison and Numbers Scepticism' (2007) Utilitas 487.

[6] Here is how Taurek puts it, supra n. 5, at 307: "For each of these six persons it is no doubt a terrible thing to die…. [S]hould any one of these five lose his life, his loss is no greater to him because, as it happens, four others (or forty-nine others) lose theirs as well. And neither he nor anyone else loses anything of greater value to him than does [the one person] should [he] lose his life. Five individuals each losing his life does not add up to anyone's experiencing a loss five times greater than the loss suffered by any one of the five."

names of the persons saved are irrelevant (call this *anonymity*) and that if you can either save one person or save *that person plus some other person* then you should do the latter (a kind of *Pareto* condition in cases where there is no real conflict of interests), then, if you think, like Taurek, that in conflict of interest situations the numbers of persons saved do not count, you are headed for intransitivity. To see this, imagine the following three choices (where the numbers in brackets to the right show the choice to save one of the three individuals from pain *p* as a '1', or not as a '0', in the order Xavier, Yvonne, Zak):

Choice A: Let Xavier and Zak each suffer from pain *p*; save Yvonne from pain *p* (0, 1, 0)

Choice B: Save Xavier from pain *p*; let Yvonne and Zak each suffer from pain *p* (1, 0, 0)

Choice C: Let Xavier suffer from pain *p*; save Yvonne and Zak each from pain *p* (0, 1, 1)

Now, according to the anonymity condition, names do not count, and so A is as good as B. (A is only a permutation of the same numbers that are in B, and anonymity requires invariant assessment under such permutations.) And, if the numbers do not count, B is also as good as C (there is no more pain suffered in C than B). But C is Pareto superior to A; in C one can save another person from pain, namely Zak, without causing any additional pain to anyone. So there is a violation of transitivity: A is as good as B, B is as good as C, but C is better than A.

This is, of course, how impossibility theorems work (more formally, of course). It seems to be impossible (in general) to satisfy *anonymity*, the *Pareto* condition, *number insensitivity*, and *transitivity*; one of these four conditions must go. Well, you can imagine that, in *normal* circumstances, Michael might have brightened somewhat at the prospect of having an air-tight logical argument for resisting Taurek's number insensitivity. This social choice theory could be useful after all! If you accept anonymity, Pareto, and transitivity, then you simply have to reject number insensitivity. That's what the impossibility theorem *says*. So, the numbers do count and maybe certain aggregations of value across persons, like aggregate utility or welfare, or (more important to law and economics) aggregate wealth, can be shown to be morally significant after all.

That would be in *normal* circumstances. However, as I have already said, I rejected transitivity, and so I was free to retain my number insensitivity. So, while Michael (gently) and others (less gently) might have wanted to suggest to me that my two arguments, the one favouring intransitivity of social choice and the other calling for number insensitivity, reflected some deep irrationality on my part, there was, at least, a kind of internal *coherence* in my position, something that was not caught by logical impossibility. If I was crazy in 1978, I was at least consistently crazy.

Indeed, I took these two arguments (the one favouring intransitivity, the other number insensitivity) quite far in 1978, further even than John Taurek himself had dared to go. For I was convinced that in a situation where the quantities of pain or, more generally, loss of preference satisfaction, *varied* across the different individuals, then we should choose so as to favour the one individual whose loss would otherwise be greatest, again without addressing the greater number of individuals who might otherwise each be suffering some smaller loss. I simply thought that the greater number of individuals should not count in this situation either. (Nor did Taurek, but I suspect that he might well have flipped a coin here too, continuing to give each and every individual an equal chance to avoid his or her particular loss.) Consider the following choice between D and E (where now I show the choices strictly as the bracketed payoffs to Xavier, Yvonne and Zak, in that order):

D: (3, 0, 0)

E: (0, 1, 2)

Here we can either choose D, and save Xavier from the loss of 3 in choosing E, or choose E, and save Yvonne and Zak from the losses of 1 and 2 respectively in choosing D. Where is the greater loss? Some might say that there is as great a loss in choosing D as choosing E; after all, in choosing D we give up the gains of 1 and 2 to Yvonne and Zak and this *adds up* to a loss equivalent to the loss of 3 in choosing E rather than D. But I was convinced (at least in 1978) that this was confusion; there can be no greater loss in some choice than what any one individual loses in that choice. Certainly no one individual can *complain* of an aggregate loss if he does not suffer it. Indeed, I shall refer to this idea that focuses on what *individuals* lose under different possible social choices as the 'complaints model', as it comes close to arguments now offered (and critiqued) under this name by such moral philosophers as Thomas Scanlon and Derek Parfit (although Scanlon does go out of his way to save the greater number when the losses are the same for all individuals)[7]. That other individuals might have additional (smaller) losses simply does not count. On this view, therefore, the greater loss (e.g., the greater pain), or the greater basis for complaint, is avoided by choosing D rather than E. In terms more reminiscent of decision theory we might say that by choosing D we minimize the maximum (individual) complaint.[8]

It is not difficult to show that the complaints model (even without invoking the Pareto condition) will violate transitivity, just as Taurek's original proposal did. But I will spare the reader a demonstration (an example appears below at the choice between alternatives N, O, and P). More interesting is the fact that the complaints model is also in some tension with the (seemingly innocent) anonymity condition. To see this, consider the following two choices F and G:

F: (1, 2, 3)

G: (3, 1, 2)

G is simply a permutation (or re-ordering) of the different individual payoffs in F, that is, it is exactly the same except that the payoffs appear for differently named individuals. According to the anonymity condition, this should not matter: F should be as good an outcome and, therefore, as good a choice as G. But, in the choice *between* F and G, Xavier has more to complain about (a loss of 2) in choosing F over G than either Yvonne or Zak can complain about (a loss of 1) in choosing G over F. *So the complaints model would have us choose G over F even though, under the anonymity condition, there is no real difference between these two alternatives for choice.* In a sense this is not surprising; the complaints model turns on *what else we might have chosen* (that is, after all, what the complaint is about), not merely on the properties (however invariant these might appear to be, say, under anonymity) of *what we do choose*. (And, incidentally, this dependence on the choice set, or partitioning of the alternatives, is also what explains the model's intransitivity as the choice set varies; Arrow famously related transitivity as a rationality condition to partition, or path, independence in the second edition of his famous book *Social Choice and Individual Values*.[9])

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[7] T. M. Scanlon, *What We Owe to Each Other* (Cambridge, Mass: Harvard University Press, 1998), at 229-41. Derek Parfit discusses Scanlon's complaints model in his paper 'Justifiability to Each Person' (2003) Ratio 368-90. For more general discussion of the complaints model, see Alex Voorhoeve, 'Should Losses Count? A Critical Examination of the Complaint Model' (London School of Economics Choice Group working Papers 2006, unpublished).

[8] The same non-aggregative idea shows up as a decision rule for individuals under the label "minimax regret". For an early axiomatization of the minimax regret rule, see J.W. Milnor, 'Games against Nature' in Thrall, Coombs and Davis eds. *Decision Processes* (New York: John Wiley, 1954). Milnor's axiomatization shows the intransitivity.

[9] Arrow, supra n.1, at 118-20. Strictly, full transitivity is more than what is required to ensure path (or partition) independence; quasi-transitivity will do. For excellent discussion of how the various collective rationality conditions

Now, so far, the choices between D and E and between F and G do not illustrate that the complaints model is in tension with more conventional social choice methods that would encourage the chooser to attend to the sum total of some good (e.g., the sum total of preference satisfaction, or of wealth, or of avoidance of pain), or the sum total of gains over losses, across persons. But this must be so since such methods are (typically) anonymous and (almost always) transitive and we have already argued that the complaints model is neither. In fact it can be shown that if the social choice method seeks to be transitive, anonymous, Paretian, and (as all such methods do, *including the complaints model*) allows for the moral commensurability of gains and losses across persons, then the method of social choice *must* be of the kind that maximizes the sum total of whatever kind of good is being measured by these numbers. That is, the method of social choice must be *additive* in the good. To see this (we now enter the world of possibility results in social choice theory, in contrast to impossibility results), consider the following two choices:

H: (1, 3, 3) (Sum Total of Good = 7)

I : (4, 1, 1) (Sum Total of Good = 6)

If the sum total of good is the indicator of preferred social choices then H should be socially preferred to I (or, in symbols, H > I). Suppose that this was not true, i.e., suppose that *not*-(H > I). It is easy to show that this will lead to a contradiction of at least one of our conditions: transitivity, anonymity, the Pareto condition, and the commensurability of gains and losses (and only gains and losses) across persons. I need a more convenient term for this last condition; so let us call the commensurability of gains and losses across persons *co-cardinality* (where *cardinality* captures the idea that the numbers can be used to measure, and give some significance to, gains and losses, and the prefix *co-* adds the further idea that the gains and losses so measured are interpersonally significant or commensurable).[10] Now consider the following sequence of paired choices:

H: (1, 3, 3)

J : (1, 1, 4)

By anonymity, J is socially indifferent to I (being a mere permutation of its payoffs). Therefore, by transitivity, if *not*-(H > I), as assumed, then *not*-(H > J). Now consider the next pair.

K: (1, 3, 1)

L: (1, 1, 2)

In K (as compared to H), and in L (as compared J), all that is changed is that (a constant) 2 cardinal units has been subtracted from the third individual's good in each case; so the gain (or loss) in moving between the two alternatives (as significant and commensurable for all persons) has been preserved. Therefore, by co-cardinality, if *not*-(H > J), then *not*-(K > L). Finally, consider the pair:

M: (1, 1, 3)

L: (1, 1, 2)

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like transitivity relate to path independence, see C.R. Plott, 'Path Independence, Rationality, and Social Choice' (1973) 41 Econometrica 1075-91.

[10] Still one of the best discussions of the informational relevance of different sorts of numbers for social welfare analysis is Amartya K. Sen, 'On Weights and Measures: Informational Constraints in Social Welfare Analysis' (1977) 45 Econometrica 1539-72.

By anonymity, M is socially indifferent to K (again, being a mere permutation of its payoffs). Therefore, if *not*-(K > L), then *not*-(M > L). But, by the Pareto condition, M > L, a contradiction. So (to avoid this contradiction under these conditions) it must be that we have started out with an incorrect presupposition and that H > I. (Q.E.D.)

It is easy to see that the same method of proof can be trotted out for *any* possible pair of choices where the total good in one choice is larger than the total good in the other. Effectively, by the repeated use of anonymity and co-cardinality, any larger total of good can be reduced, finally, to a Pareto comparison. Utilitarians, and other like-minded proponents of sum totals of the good (however construed, e.g., as utility, preference satisfaction, pleasure or the relief from pain, or wealth) as measures of social preference, will no doubt be pleased.

But, along the way to this result, the proof also shows us something that is problematic about sum total, or additive, aggregation. Specifically, the proof shows that while the moral significance of gains and losses of some good are preserved *inter*personally, or across individuals, they are not preserved in their significance *intra*personally, or for any one individual. Co-cardinality appears to begin with some basic sensitivity to an individual's gain or loss (this is what the cardinality of the numbers attends to), and then simply adds the further idea that this individually significant gain or loss needs to be commensurate with the individually significant gains and losses of other persons (so that the cardinality becomes *co*-cardinality). Indeed, this much also seems to be assumed in the complaints model, which is very much focused (in co-cardinality) on an across-person comparison of what are, nonetheless, also individually significant gains and losses (cardinality). But when co-cardinality is combined with anonymity and transitivity, this apparent sensitivity to individually significant gains and losses disappears.

We can see this if we compare the choice between H and I with the choice between H and J. Under anonymity, I and J cannot be distinguished and so must be ranked equally as social choices. So, by transitivity, whatever social ranking holds between H and I must also hold between H and J. But, *intra*personally, what is at stake for Xavier at the first bracketed position in the choice between H and I, a difference of 3 units of the good, has changed dramatically in the choice between H and J, where, for Xavier, there is no longer any difference at all. Of course, that *overall* 3 unit advantage for I has been "preserved" in J, *inter*personally, as the conversion (at the third bracketed position in the choice) of a 2 unit advantage for Zak in *H over I* into a 1 unit advantage for Zak in *J over H* (an actual preference reversal for Zak). So, in this sense, the *inter*personal significance of the cardinality has been preserved even though the *intra*personal cardinal significance of the numbers for Xavier and Zak (and even the intrapersonal *ordinal* significance in Zak's case) is lost. *It is as if there is morally significant aggregate good here that can be completely detached from, and exist prior to, the good of any individuals.* Of course this aggregate good must supervene on the good of individuals in that there cannot be changes in the aggregate good without there also being changes at the individual level. (The economic theorist captures this idea, typically, by insisting that the social welfare function, or social ordering, be a positive function of the welfare of each and every individual.) But in supervening on individual good, the aggregate good, measured in this additive way, need not make any *sense* of any individual's good.

II. Pluralism

Now the discussion in the previous section might suggest that the problem is with anonymity and transitivity. After all, the complaints model uses co-cardinality and seems to be able to attach moral significance to individual gains and losses. The 'rot' appears to set in once anonymity and transitivity are added. Certainly, this is the view that has always tempted me. Since 1978, as I have said, I have been no big fan of transitivity. But this is also where Michael's work has truly had some impact upon me. I now think that we can hang on to transitivity (subject to a scope limitation that I will make clear) and anonymity (suitably re-interpreted to preserve the intrapersonal significance of gains and losses). (Did I hear a sigh of relief?) I will even throw in the Pareto condition (again subject to a scope limitation). Further, we can also attach moral significance to gains and losses individually construed; that is, we can continue to employ cardinality. What we must give up is the prefix -- we must give up *co*-cardinality.

How is it that Michael has shown me this way forward? Not, you will (no doubt) be surprised to hear, because he sifted through all the different (im)possibility results of social choice theory, eventually pointing out co-cardinality as the most suspect normative requirement in the theorems. Indeed, I'm not really sure that Michael thinks co-cardinality *is* suspect as a requirement in the context of preference satisfaction or wealth maximization. No, the normative guidance that I took from Michael came about somewhat more circuitously.

Michael's scholarship in law and economics has always shown a commendable openness to a very broad range of normative values. One cannot read his masterpiece *The Limits of Freedom of Contract* [11]without being struck by how seriously his analysis entertains, and seeks to incorporate, values as distinct as efficiency, liberty, equality and (what some will call) the moral goods of human flourishing. This breadth of normative engagement has always endeared Michael to those who are less enthusiastic about law and economics, focused as it is, more conventionally and somewhat single-mindedly, on efficiency or wealth maximization. And Michael's leadership in this respect has helped to give law and economics scholarship at Toronto a special reputation for being more pluralistic in its approach to problems and policy, the sort of thing which others can more easily warm to.

But it has long been a part of the friendly banter between Michael and me that while Michael's pluralistic approach makes him 'warm', it has, in my view, also made him somewhat 'fuzzy' (where 'warm and fuzzy' is not always a good thing). It has never been clear to me exactly how Michael orders or regulates these different values, what it is, for example, that allows him to turn efficiency off at some point and turn liberty or equality on. This is the stuff of "tradeoffs", of course, but for me that solution always seemed to threaten the very pluralism that gave rise to it. I always wanted to ask Michael "tradeoffs in what respect?", that is, I always wanted him to reveal to me the aggregate good in respect of which these plural values were merely parts. But if they were merely parts of some prior good that ordered the terms of trade between them, then there was only the *single* aggregate good after all. The pluralism of efficiency, liberty, equality, and the goods of human flourishing was more apparent than real. On the other hand, if we maintained the pluralism, how could we order these different values which were not only external to one another, but which were also not reducible to some common space that could provide for their commensurability and rationally ordered accommodation? There seemed to be a real dilemma here: "warm, but fuzzy" or "cold and calculating, yet single-mindedly rational"?

I'm hoping that you are already sensing some connection to the earlier discussion of individually significant gains and losses of some good in contrast to an additive version of that good which has a significance for no one and which, further (as shown in the proof), in its underlying premises threatens to undermine any individually significant conception of that good. However, if that connection is still a little obscure, think of it this way. Imagine that each of the individuals, whose cardinal payoffs we have been considering up to now in the various choices on offer, is single-mindedly dedicated to some particular value, a kind of fanatical fiduciary for that chosen value. So, at some very general level, Xavier might be a proponent of efficiency, Yvonne of liberty or autonomy, and Zak of equality or fairness. (Perhaps you already know these people.) Or, for something closer to the ground, say, a policy choice or judicial decision on the regulation of electioneering signs, Xavier might be the person moved exclusively by aesthetic concerns (the signs are unsightly), Yvonne might be consumed by the free speech value (all such signs are important forms of political expression), and Zak might be the egalitarian (only small signs

[11] Michael J. Trebilcock, *The Limits of Freedom of Contract* (Cambridge, Mass: Harvard University Press, 1993)

would allow each and every voter the same cheap and accessible way to participate equally in the election). Now, given the fanaticism, the appropriate accommodation of the different views or preferences of these different individuals (the sort of exercise that we have been considering up to now) reduces to the appropriate accommodation of these quite different (plural) *values* as personified by these different fanatical fiduciaries. Drop the intermediate personification step and the accommodation of the different payoffs to the different persons just *is* the accommodation of different, or plural, values.

Admittedly, we have come a long way from Taurek's idea that there is no (greater) real pain in an aggregate of pain that no one individual suffers, but the concern at this point seems to be very similar. What could be the one aggregate value that, generally, exists prior to efficiency, liberty, and equality, and preserves the significance of each, or, more specifically in the case of the electioneering signs example, that exists prior to aesthetics, free speech, and equal political participation, and makes sense of each of these concerns as a value to be addressed (on its own terms) in the regulatory or judicial decision? If, like me, you think that there is no such aggregate value (just as there was no such aggregate pain), then you have the connection to the earlier discussion that we need. What is required in the accommodation of plural values, just as much as what was required in the accommodation of some good (or bad) viewed (suffered) individually, is a method of aggregation that *preserves* that plurality of values, without reducing one value into another or without lumping all of them together into some common space (like utility) that denies their plurality. I want now to suggest that what is needed is aggregation without co-cardinality. If we can secure that (and secure a version of anonymity, transitivity, and Pareto as well), then I think we can secure Michael, not only as "warm and fuzzy", but as "warm, fuzzy (now in that good sense), *and* rational".

How might we do this? We could begin by recognizing that not all forms of aggregation need to be *additive*. It is true that most forms of interpersonal aggregation that we typically meet (e.g., utilitarianism, cost benefit analysis, wealth maximization, etc.) are additive. That is, they begin with cardinally significant measures of the different individual values (utilities, preferences, measures of willingness-to-pay) and then add up the gains and losses of these values as we move between the alternative choices. But suppose that instead of adding up the different numbers representing the payoffs for any individuals in any given social state we *multiplied* them instead, and then set our social choice rule as choosing that social state for which this *product* was a maximum. As John Nash demonstrated sixty years ago, this social choice rule has some very interesting properties. [12] Most important for our discussion is the fact that the multiplicative aggregation rule can allow the numbers to have cardinal significance for each individual (in the interpersonal context), or for each value (in the context of pluralism), so that we can meaningfully compare gains and losses between different choices *for that individual or value*. Yet it can do this without this cardinality having any significance as *co*-cardinality, that is, as significant or commensurable *across persons or values*.

To see this, consider the following three choices N, O, and P:

[12] J.F. Nash, 'The Bargaining Problem' (1950) 18 Econometrica 155-162. Strictly speaking, the product rule attends to the product of any *gains* above some arbitrarily chosen zero point (in bargaining Nash identified the status quo as a natural zero point for all bargainers); so, unlike for the additive rule, there is some sensitivity in the product rule to changes in this zero point for some individuals, or values, and not others, i.e., some sensitivity to the addition or subtraction of a constant from the numbers representing the payoffs to a given individual or value. Nash's bargaining solution satisfies a condition like anonymity (which he calls symmetry) , a condition like transitivity (which in Nash's solution appears as choice consistency condition on shrinking opportunity sets, something that in the social choice literature is now called "alpha rationality"), the Pareto condition, and cardinality (but not co-cardinality). For an accessible discussion of the Nash solution as a social choice rule, in comparison to other social choice rules like utilitarianism and Rawls' leximin rule, see Donald Wittman, 'The Geometry of Justice: Three Existence and Uniqueness Theorems' (1984) 16 Theory and Decision 239-50.

N: (6, 10, 2)

O: (1, 10, 9)

P: (8, 4, 3)

The first thing to note is that the additive or sum total aggregation rule would choose O as the best alternative, the multiplicative or product aggregation rule would choose N as best, and the individual complaints model (which minimizes the maximum individual complaint) would choose P. So, each of the social choice rules offers a quite different recommendation, reflective of the quite different properties of each rule. For example, the intransitivity of the complaints model is nicely illustrated here: if the alternatives were presented for pairwise comparison, it would choose O over N, P over O, but N over P. Also, that it would choose P over N from the triple, but N over P from the pair is also thought by some to be odd; why should the addition of O as an alternative reverse the choice between N and P? (The answer, of course, is that the addition of O gives the third placed individual in the brackets, Zak, something to complain about in the choosing of N, a complaint large enough to move us to choose P under the model.)

But I want to emphasize how co-cardinality has a role under the additive rule which it does not have under the multiplicative rule even though we can insist that the numbers are cardinal throughout for each individual or value. We have already seen above (in comparing the choice between H and J with the choice between K and L) that we can preserve co-cardinality (or the significance of gains and losses between different choices for different persons or values) if we simply add or subtract a constant to each of the payoffs for a given individual across the different choices, leaving the payoffs of all the other individuals the same. Adding or subtracting such a constant raises the sum total of each of the choices by the same amount and, therefore, does not affect their relative ordering. We might say that the additive or sum total aggregation rule is invariant in its recommendation with respect to the addition or subtraction of such a constant and, therefore, treats such a change in the numbers as morally insignificant. (A sum total rule, like utilitarianism, is not so much interested in *how happy* people are, something that is changed, of course, by the addition or subtraction of a constant, as it is in *how much happier* they are in some choices as compared to others, something that is not; this explains why utilitarianism is not much good at doing distributive justice, which typically needs social choice to be sensitive to *levels* of happiness or welfare, not just gains or losses.[13])

However, the same additive rule would *not* be invariant to a multiplication of some one person's cardinal numbers by some constant, say those of Xavier in the first bracketed position. Suppose, for example, that we simply doubled all the payoffs for Xavier for each of the three alternatives N, O, and P, leaving the payoffs of the other individuals the same. This would result in the following array of payoffs for the three choices (which we re-label here with an asterisk):

N*: (12, 10, 2)

O*: (2, 10, 9)

P*: (16, 4, 3)

Now, because Xavier's numbers have been doubled (but nothing has been changed for the other two individuals), so have the gains and losses between the different choices for Xavier (but, again, without changing anything for the other two individuals). So the interpersonal significance of gains and losses has

[13] Sen, supra n. 10, at 235.

not been preserved, something that is enough to change the sum total or additive aggregation rule's recommendation from what it was before, namely O, to a new choice N*. In other words, the additive rule, because it is co-cardinal, is not invariant to such changes; it lends moral or interpersonal significance to (and varies with) these sorts of changes.

However, there is invariance of social choice (with respect to a multiplication of some one person's cardinal numbers by some constant) under the multiplicative aggregation or product rule. Where this rule would have chosen N in the first array, it continues to choose N* in the new array. The reason for this is easy enough to appreciate: multiplying Xavier's payoffs by a constant (here by 2 under a doubling of his payoffs) simply multiplies the product across each row, or the product of the payoffs for each and every social choice, by the same number. The relative ordering of the social choices is unchanged. So the invariance of choice under the multiplicative aggregation rule (in contrast to the change we observed under the more conventional additive rule) represents the insensitivity of this rule to any claim (by a person or value) to the significance of *co*-cardinality. The cardinal values (or the gains and losses), while significant for a person or value, have no (commensurable) significance across persons, or values, at all. And this, I want to suggest, is exactly the sort of aggregation rule we want, at least if we want to make proper sense of gains and losses of pain as suffered by individuals (about which Taurek was, at least in part, so worried), and of the genuine plurality of very different values (to which Michael's work has always shown such a commendable sensitivity).

III Proportionality

Now, all this may sound a bit obscure and remote from the sorts of things that lawyers and legally trained academics take an interest in. However, I want now to argue that something closely related to the multiplicative aggregation rule is very much alive within the law under the guise of proportionality review. This is the sort of review that Canada's own Supreme Court has engaged in since *R*. v. *Oakes*[14] when Charter protected rights are limited by (or, as some might be tempted to say, "traded off against") pressing and substantial state interests under section 1 of the *Canadian Charter of Rights and Freedoms.* And, of course, as Michael's close friend and colleague David Beatty has shown so convincingly in his book *The Ultimate Rule of Law*, the language of proportionality increasingly characterizes the jurisprudence of a broad range of jurisdictions. [15]

Think first about what the term proportionality means to a lay person. Suppose that I was to drain your full water barrel of 30 percent of its contents and that you were to drain mine of 50 percent of its contents. While you would have drained mine of a higher proportion of its contents than I would have drained yours, we would not be tempted to say anything at all about the relative quantities of water that had been drained from each barrel. My barrel might have lost half of its contents, but be very small as compared to the size of your water barrel. If that were true, then I might well have lost much less water

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[14] [1986] 1 SCR 103, paras. 60-70, *per* Dickson CJ. For a nice logical analysis of the various steps in *Oakes*, see Denise Reaume, 'Limitations on Constitutional Rights: The Logic of Proportionality' (University of Oxford: Legal Research Paper Series, August 2009)

[15] David M. Beatty, *The Ultimate Rule of Law* (Oxford: Oxford University Press, 2004). Robert Alexy is probably the best known proponent of proportionality analysis; see his *A Theory of Constitutional Rights* (Oxford: Oxford University Press, 2002).

than you even though (still) I have lost a higher proportion of mine. Proportionality assessments are simply invariant to changes in the relative size (or commensurability) of our two barrels.[16]

What can be said (and, just as important, *not* said) of the proportional draining of barrels can also be said (and not said) of the proportional limitation of rights and state interests within proportionality review. It is just a mistake to liken this sort of mutual accommodation of rights and state interests to a kind of additive cost-benefit analysis, or utilitarianism, where the thinking conventionally is that, under proportionality review, we are getting to some greater sum total of good, all things considered. As the barrel example suggests, proportionality comparisons do not allow us (and, under percentages, would not even tempt us) to say *any* such additive things. But that limitation need not hamstring us either. For under proportionality analyses we can make use of non-additive, non-co-cardinal social choice rules, the sort of thing that makes much better sense of (and does less conceptual violence to) the very pluralism that sets up the need for such an exercise in mutual accommodation in the first place. To see this, let us reconsider one final time, our last three choices N, O, and P.

First, it is worth stating explicitly that the numbers represented in the array of choices N, O, and P, if they are cardinal but not co-cardinal, can just as easily be represented as the percentages or proportions of some specific good (an individual's satisfactions or a particular plural value). For remember that we can preserve that cardinality, although not the *co*-cardinality, under any transformation of the numbers (for each individual or for each value) that either simply adds or subtracts a constant a, or multiplies the numbers by the same constant b, or both. (This is what the economist means by a linear transformation, $a + bX$, of the series of numbers X, the sort of transformation that preserves the cardinal significance of the representation, or the ratio of the differences, or gains and losses, in the numbers for a given individual or value.) Suppose, for example, that Xavier's payoffs in N, O, and P represented, with cardinal significance, the range of the very best and the very worst that he, as a fanatical fiduciary, could achieve for some value, say liberty, in social choice. P is best, O is worst, and N is somewhere in between. Where in between? What proportion of the overall liberty scale from worst to best is satisfied by alternative N? The transformation to proportions or percentages is easy[17] and results in alternative N representing just over a 70 percent satisfaction of the liberty value on a scale that has been (linearly) transformed into a scale ranging from zero percent relative satisfaction of liberty (at alternative choice O) to one hundred percent relative satisfaction of liberty (at alternative choice P) .

Of course, it is unlikely that in any real choice situation we would be entertaining the thought, in one of our alternatives, of choosing the worst possible outcome for some important value, say, liberty. Nor is it likely that one of the alternatives available is the best possible outcome for liberty. So in such situations we would not really have alternatives in play that would show values 0 and 100 on our proportionality scale. Much more likely all the alternatives for choice would show values "in between", like the alternative N did at a level of 70 percent satisfaction for the value of liberty. So, all the choices would show positive (but less than 100 percent) proportionality for each of the different values. And to rank the possible social choices we could simply multiply these different values together in the way already suggested by the multiplicative aggregation or product rule, confident in the fact that this rule preserves the individual significance of the cardinality of each value and (now) encouraged by the fact

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[16] My colleagues at the University of Toronto Faculty of Law will be relieved that I did not, yet again, use the example of proportionality that I believe operates when a judge determines which dog is the "best in show" at a dog show involving many incommensurable breeds; for discussion of that example, see Bruce Chapman, 'Law, Incommensurability, and Conceptually Sequenced Argument' (1998) 146 University of Pennsylvania Law Review 1487, at 1492 n10.

[17] It involves solving (for the two unknowns a and b) the two equations $0 = a + b1$ and $100 = a + b8$, and then substituting these solutions into the equation $Y = a + b6$.

that these are the sorts of numbers that lawyers and legal academics have become accustomed to using (even if only implicitly) under the idea of proportionality review. [18]

But there is a residual ambiguity in the term proportionality that needs to be addressed. While the multiplicative aggregation or product rule preserves the idea of a proportional (cardinal) satisfaction of individual values (and does not do violence to that idea in the way that an additive rule would), there is an additional sense of proportionality that the lawyer or legal academic would worry about in the use of the product rule. It is possible, for example, that the product of these percentage or proportional numbers across all the different values could be quite high for some alternative social choice even though the proportional achievement for a *particular* value in that choice was quite low. And the lawyer or legal academic would be tempted to add that this meant that this value was being denied in a way that was "out of all proportion" to the others. This suggests that not only is there proportionality significance *in* the numbers (the point I have been developing so far), there is also a further significance in the equal or proportional accommodation *of* these different numbers (for these different values) in what we choose. While the product rule is sensitive to the first interpretation of proportionality, it is not particularly good at accommodating the second. And the latter, arguably, is a lot of what a case like *Oakes* is all about.

Could it *only* be about that? Could it be that proportionality review is just about the equal relative achievement of the different values and that all my previous analysis, inspired by Michael's commendable commitment to pluralism, is simply beside the point? I hope that enough has been said by now to convince the reader that this cannot be so. What is required is a social choice rule that preserves what is significant *in* the different values that are being aggregated, something that an additive approach to these values simply cannot do. This is what I so dimly saw way back in 1978 when, in a different aggregative context, I teased Michael with the John Taurek problem. A multiplicative aggregation or product rule *can* do what is required, although it might fall prey to worries about the second kind of proportionality that reaches *across* the different values. But we must be careful, as we unpack what proportionality means in this second sense, to preserve proportionality in the first sense. Otherwise, just as in the additive models of aggregation so much on offer, we will be in danger of accommodating plural values in a way that really makes no sense of the gains and losses that are in play *for those values* as we move between different possible social choices.

I indicated earlier that I would come back to the problem of intransitivity before finally letting you go. You will recall that there was a logical connection between my earlier number insensitivity and my support of intransitivity in social choice. However, as I hope my discussion shows, I have refined my number insensitivity over the years. The product rule, for example, will obviously be sensitive to the number of positive values being incorporated into the multiplication; adding a positive value, or adding to a positive value, *will* count. And, of course, the product rule (consistent with the fact that it is number sensitive) will generate a transitive social ordering of the alternatives available for choice. So, does this mean that I am also cured of my somewhat peculiar commitment to intransitivity?

No, not really. I am committed, of course, if I use the product rule, to transitivity in my accommodation of proportional values at the *final* stage of social choice. The alternatives considered at that stage of social choice might well be transitively ordered. But, as I have tried to suggest in another paper,[19] I think a final stage proportionality review can represent the perfect completion of the earlier

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[18] For a thorough and creative discussion of Nash's product rule and its possible relation to proportionality review, see Paul-Erik Veel, 'Rational Legal Decision-Making Among Incommensurable Options' (Directed Research Paper, Faculty of Law, University of Toronto, April 2009).

[19] Bruce Chapman, 'Defeasible Rules and Interpersonal Accountability' forthcoming in J. Ferrer & G.B. Ratti eds. *Essays in Legal Defeasibility* (Oxford: Oxford University Press 2010), at n. 29.

process of multi-staged social choice which we observe in legal adjudication and which is so obviously path dependent in its structuring of this final choice across the remaining social alternatives. In adjudication, the different parties offer a sequenced staging of claims, responses, and rejoinders that eventually lead us down a particular path to some final result. In doing this, the parties become hostage to each other, and the claims and arguments that each chooses to make against the other, as they come to fashion the final result as a matter of their shared, interactive, or collective rationality.

Such a collectively rational decision process is not likely to give rise to a transitive ordering of all possible social choices in the way that Arrow and other social choice theorists tend to presume. It will matter to the selection of the final result what particular path the parties choose to go down. Alternatives that the parties might have ended up choosing will not be chosen because they were put in issue early in the process (just because that is how the argument happened to go) and eliminated. But, while the result will be path dependent, it will not be arbitrarily path dependent in the way that so concerned Arrow and which convinced him to require transitivity. Rather it will be a result that reflects the actual claims that the parties chose to make, and the actual responses to those claims (and the rejoinders to those responses) that the parties chose to offer in reasoned reply.

In this respect too, the parties will have been held rationally accountable to *each other*, what Steven Darwall has called “second person” accountable. [20]They will not so much be responsible to how the world is as a “third personal matter”, that is, as a matter independent of the claims and replies they choose to make. And this is the connection to proportionality review. Just as proportionality review is about the mutual accommodation of plural values that remain external to one another and free from reduction to some third (independent and prior) value in respect of which they are made commensurable, so the interaction between parties in adjudication involves the mutual accommodation of values advanced within a (path dependent) sequence that is significant because it is the sequence that the parties construct for themselves and for each other. Proportionality review, at least if there are claims and defences that are not completely answered (and negated) by the other side, is exactly the right way to finish off such a sequenced adjudication.

By contrast, think how bizarre it would be to finish such a sequence with a utilitarian form of balancing (any additive form of aggregation would do) that reduced each party’s claim to a measure of utility. If utility is important enough at the end of this process to be the final arbiter of the parties claims and counterclaims, one wonders why, as an independent good, it was not sufficiently important to run the show from the beginning, that is, completely independent of the claims and counterclaims that the parties themselves might choose to make. In contrast to proportionality review, an additive aggregation approach, of which utilitarianism is only one example, can make no sense of the actual sequence as the parties have chosen to have it unfold. In this respect, therefore, there is still a close connection between my support for intransitivity (and the path dependence that transitivity makes possible) and proportionality.

So Michael might be disappointed that the guidance I took from him, and from his openness to plural values, has not led me completely clear of intransitivity. As you might have expected from someone burdened with such a peculiar view, it seems that I have cycled back to that position, the position I originally held on this matter in 1978 when we first met. Perhaps this too is a path dependent result for me. But, as it is a path (of claims and counterclaims) that I have shared with Michael as a colleague and friend for over 30 years, this is a path dependency of result that anyone, even Kenneth Arrow, must surely envy.

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[20] Stephen Darwall, *The Second-Person Standpoint: Morality, Respect, and Accountability* (Cambridge, Mass: Harvard University Press, 2006)